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ABSTRACT

During "lesson study" teachers formulate long-term goals for student learning and development, collaboratively work on "research lessons" to bring these goals to life, document and discuss student responses to these lessons, and revise the lessons in response to student learning. This document summarizes the content of a symposium to discuss the relationship of lesson study and teachers' knowledge development. Panelists presented models, methods, and examples of lesson study and discussed capacity development through lesson study. Three examples of lesson study in action are described. One involved a third-grade lesson study group. The second example contains meeting notes, transcripts, teacher interview results, and student mathematics interviews from a kindergarten lesson study group with an outside mathematics specialist. The third example describes videotapes and transcripts from an ad hoc lesson study group that planned, taught, and revised a research lesson during a 2-week summer workshop. (SLD)

Lesson Study and Teachers' Knowledge Development: Collaborative Critique of a Research Model and Methods

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April 2003

Lesson Study and Teachers' Knowledge Development: Collaborative Critique of a Research Model and Methods

Interactive Symposium Outline

AERA 2003, Division K, Section 7, Tuesday April 22, 2003

Session Panelists: Catherine Lewis, Rebecca Perry, and Aki Murata, Mills College;

Session Chair: Brian Lord, Education Development Center;

Session Discussant: Akihiko Takahashi, DePaul University

Symposium Goals

1. Learn about and provide input to a conceptual framework designed to represent lesson study's relationship to teachers' knowledge development;
2. Evaluate the adequacy of early evidence of teachers' knowledge development during lesson study and of methods for capturing it;
3. Strengthen connections among researchers interested in studying the impact of lesson study and other professional development approaches.

Symposium Format

Introductions (Brian Lord, EDC)

Part 1: Presentation of Model, Methods, and Examples (Perry, Murata, Lewis)

- Presentation of current models and measures (8 minutes)
- Presentation of 3 examples of evidence (27 minutes)

Part 2: Feedback and Discussion on Models and Evidence

- Written feedback from all participants (10 minutes)
- Discussion with audience (35 minutes)

Closing Synthesis: Discussant Akihiko Takahashi, DePaul University) (5 minutes)

Background on Lesson Study and Teachers' Knowledge Development

During "lesson study" teachers formulate long-term goals for student learning and development; collaboratively work on "research lessons" to bring these goals to life; document and discuss student responses to these lessons; and revise the lessons (and the broader approach to instruction) in response to student learning (Lewis & Tsuchida, 1998; Lewis, 2002; Fernandez et al., 2001; Yoshida, 1999; Stigler & Hiebert, 1999). These four activities – planning, observing, analyzing student learning, and revising instruction – constitute a complete cycle of instructional inquiry, making the lesson study process consistent with many of the qualities U.S. researchers suggest are "effective" for teacher professional development (Darling-Hammond & McLaughlin, 1995; Goldenberg & Gallimore, 1991; Little, 1981; Loucks-Horsley et al., 1998; Putnam & Borko, 2000). Indeed, some researchers argue that lesson study is exactly the type of teacher professional development process that can help build a usable knowledge base for teaching (Hiebert, Gallimore, & Stigler, 2002).

The perceived effectiveness of lesson study by practitioners has led to its rapid spread to schools and districts across the US. (Lesson Study Research Group; Stepanek, 2001; Takahashi, 2001; Council for Basic Education, 2000; Germain-McCarthy, 2001). Teachers report being drawn to lesson study because it engages them in the selection, study, and solution of a problem that is important to them. Rather than relying on external measures of validity (e.g., standardized test scores) or on controlled experimental studies designed to build widely generalizable results, lesson study relies on teachers' close-up study and documentation of student learning, engagement, and behavior during actual classroom lessons at their own schools.

The conflict between lesson study – often experienced and expected by practitioners to produce useable results to improve their teaching, but not yet proven effective by "scientific" research – and President Bush's Leave No Child Behind reform agenda – calling for programs based on sound, scientifically-based research – is immediately apparent. The increasing popularity of lesson study raises the question: How will we know if lesson study is improving learning outcomes for teachers and, in turn, for students?

Part 1 How do teachers develop knowledge during lesson study? Presentation of Models, Methods, and Examples.

Theoretical Model and Methods (8 min):

The figure "Capacity Development Through Lesson Study", adapted from D. Ball (2001), represents our current thinking about how teachers use lesson study to build capacity for instructional improvement. "Capacity," consists of three types of resources: knowledge for teaching¹, knowledge of lesson study, and motivation/ efficacy. Four system features interact to build capacity:

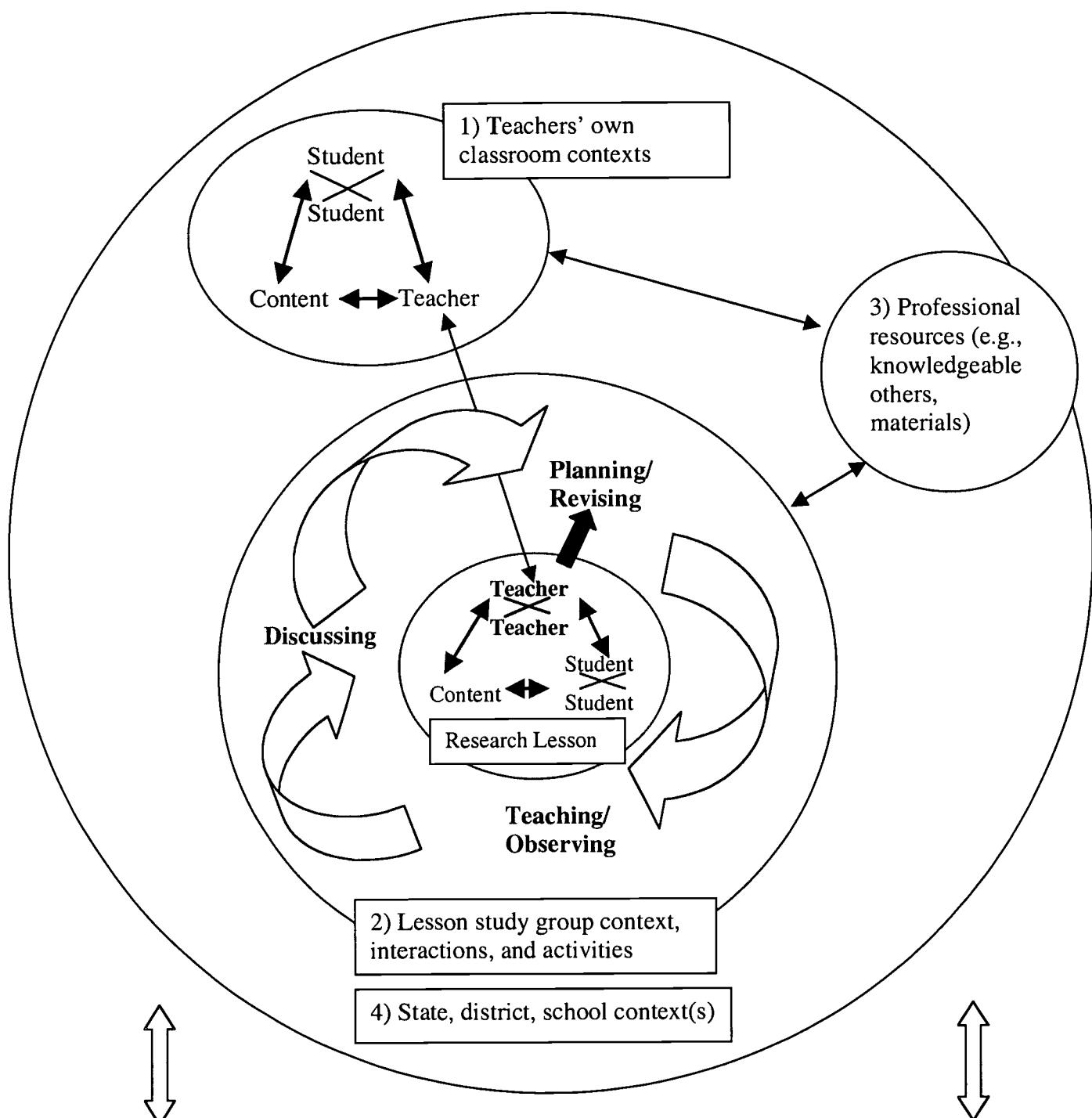
- 1) The teachers' individual beliefs, knowledge, and classroom contexts;
- 2) The lesson study group's context, interactions and activities.;
- 3) The available professional knowledge resources (e.g., print materials, knowledgeable others); and
- 4) The contexts (state, district, school) in which the lesson study work takes place.

Two-way arrows between capacity and the system features are an important feature of the model. Capacity is conceived both as an outcome of lesson study and as an input to further development of lesson study activities.

The research lesson (shown in the centermost circle) is seen as the catalyst for repeating cycles of lesson study activities – planning, observing, discussing, and revising. The research lesson is the result of three-way interactions between students, teachers, and

¹ RAND Corporation (2002). <http://www.rand.org/publications/MR/MR1643.0/MR1643.0.pdf>

Capacity Development through Lesson Study



content. As in regular classroom contexts, student-student interactions are seen as a central contributor to the lesson outcome. Unlike regular classroom contexts, teacher-teacher interactions are also a central contributor to the lesson outcome. For example, in the planning phase of lesson study, teacher-teacher interactions about content may generate new pedagogical approaches to be included in the research lesson; in the discussing phase, teacher-teacher interactions about student conversations may generate new approaches to be included in a subsequent research lesson.

Three major bodies of literature inform our model: research on teacher knowledge and change, research on reform implementation and change, and research on professional development. The framework, along with those developed by other researchers studying the issue of teachers' knowledge development is posted at <http://www.lessonresearch.net> . A reference list from these bodies of literature is provided at the end of this handout.

Examples to Catalyze Discussion of “Evidence” and Model (9 minutes each, total of 27 minutes)

Three examples are presented from one medium-sized elementary/ middle school district, which began lesson study during the summer of 2001. The district serves middle- and working-class families in a suburban and small urban setting; most lesson study activity focuses on mathematics (some language arts), and the founding leaders of lesson study included three math coaches and a project coordinator (two of these four, who continue to lead the project, are half-time classroom teachers). Teachers participate in lesson study on a voluntary basis and receive a stipend of \$500. The number of teachers participating has increased from 28 to 78 over the past two years. The examples are selected to illustrate the wide range of lesson study activities, data collection methods, and outcomes (types of capacity) that might be studied even within a single setting.

1. Example One: Meeting transcripts from a third-grade, school-based lesson study group. (Rebecca Perry)
2. Example Two: Meeting notes, transcripts, teacher interviews and student mathematics interviews from a kindergarten lesson study group with outside math specialist. (Aki Murata)
3. Example Three: Videotape and transcripts from ad hoc lesson study group that planned, taught, revised, and re-taught a research lesson during a two week summer workshop. (Catherine Lewis)

Part 2: Feedback and Discussion on Models and Evidence

1. Written feedback from audience (10 minutes) (See response form)

Written responses will be tabulated and/ or transcribed and, sent to interested attendees who provide email address on sheet circulated.

2. Discussion with audience moderated by Brian Lord (35 minutes) 2-minute limit for individual comments. Each of the examples and the model will be discussed in turn (5-10 minutes per topic). Discussion starters:

Related to Examples:

- How convincing was the evidence of teachers' learning? What is most problematic?
- What additional types of evidence would convince you that teachers are learning through lesson study?
- What are the strengths and shortcomings of the methods presented? What other research activities should be considered?

Related to Model:

- To what extent does the model represent your understanding of how teachers learn during lesson study? What revisions would you suggest?
- Does the evidence fit the model? What changes to the model are suggested by the evidence?
- In your opinion, what is the seminal literature about teachers' learning and knowledge that should inform lesson study research?
- What knowledge development "outcomes" are most relevant to lesson study, and should they be considered in sequential stages?

Comments and Discussion Synthesis, 5 min.: (Akihiko Takahashi)

Closing comments will be provided by Dr. Akihiko Takahashi, who has more than twenty years of first-hand experience in lesson study in Japan and has more recently been active in lesson study in the US.

Participants who sign circulating sheets will receive a database of all attendees and a summary of their lesson study activities and/ or research.

Please leave your written reflection sheets. We will be happy to return a copy to you if you so note and provide a mailing address.

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Evidence of Teacher Learning, 2002-2003 school year Example 1

School Context: The school is in its 2nd year as a school-wide lesson study site. This year is the first year that group composition has been determined by grade level, a change brought about by the principal's interest in using lesson study to understand and help teachers implement a district agenda of standards-based instruction. Teachers are asked to select a particular standard to work on during the year and to organize their work according to a year-long agenda. The agenda is designed to balance the cycle of lesson study activities to include more time for reflection on what teachers learned about the standard.

Group Context: The group consists of five 3rd grade teachers. Three teachers worked together last year in what was considered by them an unsuccessful attempt at lesson study, deemed so because the work focused on creating a lesson from scratch and because the group had difficulty with group norms (e.g., not showing up at meetings on time, not committing to the work). Teachers' experience ranges from 2 to 25 years. Three group members are in their second year of lesson study; one member is in her first year; and one member (also a half-time math coach) is in her third year of lesson study.

Lesson Study Topic: The group selects California Mathematics Standard, Grade 3, Number Sense Standard 2.8: "Solve problems that require two or more of the skills mentioned above (calculate and solve problems involving addition, subtraction, multiplication, and division)." The group interprets this standard as one focused on two-step (they later decide that it can be more than two steps, or multi-step) problem-solving. They select the standard because standardized test scores reveal that students are weak in number sense and because multi-step problem solving is a mathematical concept unique to the third grade standards. Through their research, the group identifies a strategy for helping students learn problem solving from the *Everyday Math* curriculum. The process includes five steps, which students are to complete to be successful problem solvers:

1) What do you know [about the problem]? 2) What do you want to find out? 3) What will you do? 4) Answer the question – Can you write a number sentence to show what you did? 5) Check to see if the answer makes any sense. How do I know?

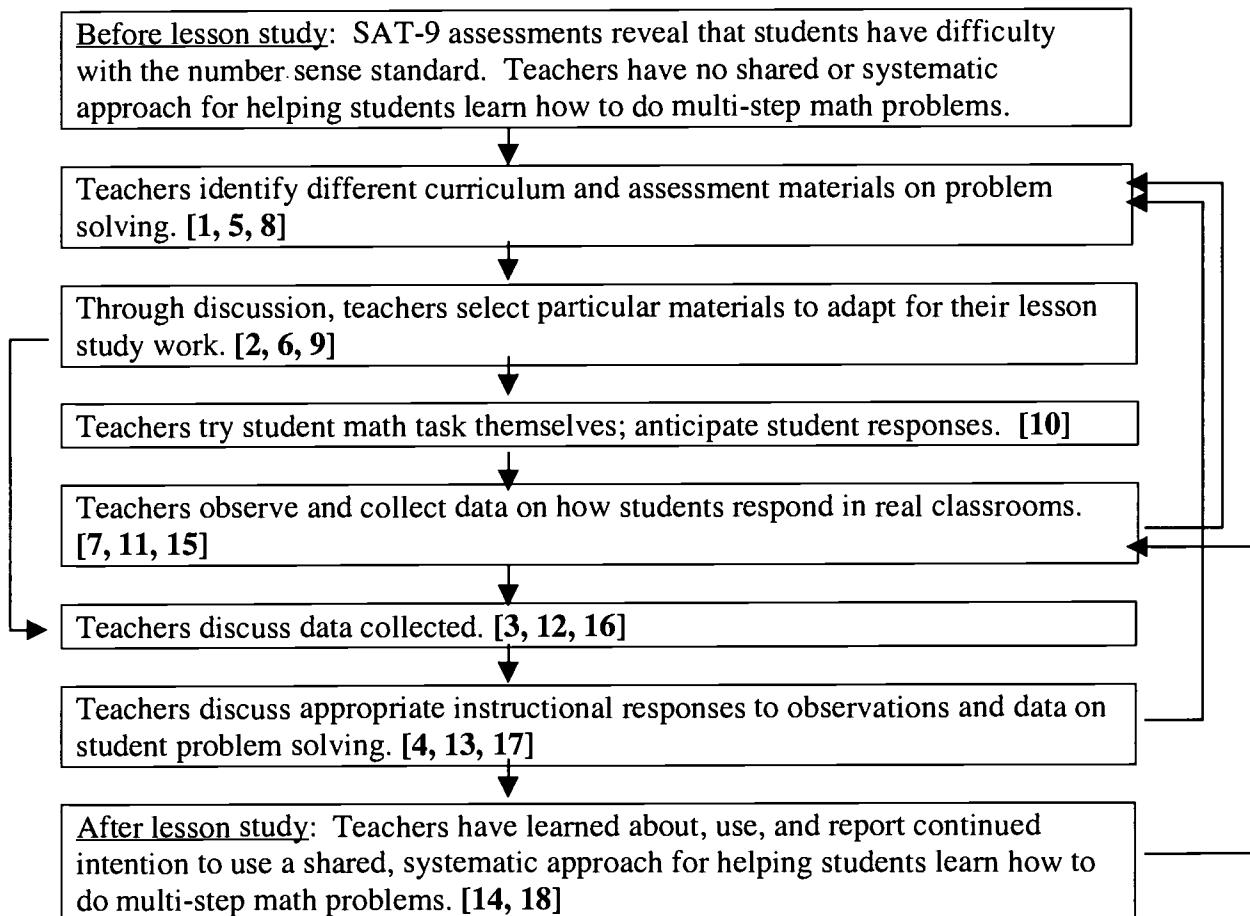
Data Collected: Individual pre and post lesson study interviews with all group members (post interviews not yet scheduled); observations of all lesson planning, debriefing, and reflection sessions – written field-notes (audio-tape back-up and transcript when possible); collected artifacts - teacher reflection forms, student work, math tasks; transcript of research lesson.

Strengths and weaknesses of data presented: The data presented here illustrate what teachers learn about problem solving through their lesson study work. The strengths and weaknesses of the data seem to be two sides of the same coin: reliance on discussion transcripts. While use of the teachers' actual words enables us to observe the progression in their thinking, the transcript data limit us to reported use or intended use of the

problem-solving guide as opposed to observed use. Additionally, the data reveal *opportunities for learning* – instances where learning might occur – rather than measuring *if or how much learning* occurs.

The data reveal that teachers had opportunities to learn about various difficulties students have with problem solving, and that teachers located, tried, and decided to integrate into their practice an approach to teaching problem-solving. Is the group's solution – to teach the problem-solving process and integrate it earlier into the next year – mathematically useful learning that will enable these teachers to teach for understanding and to help students achieve the standard?

Opportunities for Teachers' Learning about Problem Solving through Lesson Study²



² The numbers in brackets refer to data presented in tabular format on the following pages.

Ref #	Data (presented chronologically)	Researcher notes ³	Date, source
1	Meeting minutes summarize the group's agreement in the October meeting to "...come [to the next meeting] with a possible lesson we could re-vamp. This lesson should address a 2-step problem. [Teacher 19] handed out examples of the MAC test for a possible starting point."	<i>Teachers are asked to access materials for a specific purpose and come to the next meeting prepared to tell others about the material [kt]. Teachers illustrate desire to adapt rather than create a lesson [ls]. Teachers use meeting minutes to record their decisions [ls, me].</i>	<i>Nov 02, Field-notes, meeting minutes</i>
2	Teachers select one of three assessment tasks brought to the meeting to administer in their classes to "throw it out there to see who knows what." The group chooses the least difficult task, feeling that the other two more difficult tasks could be used for the lesson and the post-assessment. Teachers agree to collect data in their classes, and bring the data to the next meeting to discuss.	<i>Teachers begin with assessment to see what students already know about their topic [kt, ls]. Teachers discuss qualities of a useful pre-assessment [kt]. Teachers value classroom data on student understanding [kt, ls].</i>	<i>Nov 02, Field-notes</i>
3	Teacher 111: I was surprised. My students didn't really know how to approach the problem, and what to do. ...they don't know how to verbalize their thought process... Teacher 62: [Reporting on her class] They had many different ways of figuring it out [lists strategies]... So, I think they understood what strategies to use at the beginning... Teacher 19: [Reporting on her class] I thought that a number of them had the right answer, but when I looked at what we talked about we wanted kids to be able to do, like 'show no fear. Have confidence. Use a variety of strategies. Show perseverance. Be organized. Organize and communicate how they feel and use mathematical vocabulary.' Those are things I'd want to target in the instruction....	<i>Teachers gather data that illustrate that students in different classes have different difficulties with problem solving, expanding their understanding of difficulties that 3rd grade students have with problem solving [kt]. The difficulties seem to be in steps 2 and 3. Teachers relate their data on student understanding to their goals for students' development [ls].</i>	<i>Dec 02, Transcript</i>

³ The following codes for increased capacity (as interpreted by researcher) are used: kt – knowledge for teaching; ls – knowledge of lesson study; me – motivation/ efficacy.

4	<p>Teachers discuss one particular strategy from material they gathered – the five step process.</p> <p>Teacher 19: How do you then help those kids understand how to record it in an organized way? ...Not just get the right answer, but be able to do a really efficient representation of what their thinking was and how they got it. I would really hope that by 3rd grade, kids would do numbers and not have to draw a picture.... What I'm wondering is, sometimes when you have kids write a paragraph or a news article, you say you have to have the who, what, where, why, when. That's a way to go back and make sure that you have the who, what, where, why, when in there. So, is there something like that that you can tell kids about when you've explained the problem, you have the, it wouldn't be the who, what, when, where, why for a math problem, but what would it be?</p> <p>Teacher 69: Almost like a checklist... [break in transcript] In this particular book... it follows the same thing in terms of problem solving strategies.... In each one, it says... "What's the information? What's the question? Draw a picture. Choose an operation. Do the operation and answer the question."</p> <p>Teacher 19: ...At this point we're talking about helping... kids solve a... multi-step problem, right? So, we're talking as part of the instructional strategy to teach kids to focus on "what's the information," "what's the question," and it seems like you want them to think about what's the most efficient way to figure it out... One way to think about the lesson study lesson, too, is like we create a hypothesis that we have about how we can make kids more successful at attacking multi-step problems. So we come up with a hypothesis that if you do this, kids are going to be more successful and move closer to success with the standard, you know? So, we're kind of kicking around the "if you do this" part.</p>	<p><i>Teachers discuss difficulties they identified and one strategy for addressing these difficulties [tk].</i></p> <p><i>Teachers discuss their expectations for 3rd grade students in terms of particular strategies they would want them to use [tk]. Teachers hear that lesson study is about hypothesis-testing [ts].</i></p>	<p><i>Teachers revise the criteria they use to identify and review materials based on new shared understanding [tk]</i></p>
5	<p>Prior to January meeting, two teachers reviewed more material to identify a problem-solving lesson.</p>		<p><i>Teachers revise the criteria they use to identify and review materials based on new shared understanding [tk]</i></p>

6	<p>Two teachers report their observations about the material to their colleagues:</p> <p>Teacher 67: The thing that really stuck out to me... was that so much of the learning takes place in the sharing. So many of the problems have multiple ways of attacking them, and the lesson itself might be in looking at the various strategies...</p> <p>Teacher 19: [Agrees that the article focused on kids sharing strategies.] One difference, though, I think... I think what's the same about all of the problem solving stuff that we looked at, was that there are the five steps, and within those five steps, there is a choice that you make about how you're going to solve it. And one of the points that that chapter makes is that you won't necessarily use all five steps every time... – maybe you jump in at 3, or you just use 3, 4, and 5. But even if you know the answer right away, you would at least use one of those steps – the checking your answer.... And when I was looking [for the lesson], I was kind of keeping in mind those five steps, and from reading this article, the thing that jumped out to me was the importance of having kids be able to create a model that represents the problem, to figure it out. So the lesson I copied is a lesson on teaching kids how to make a model for story problems... I picked up on the modeling thing, because to me it seemed like when we looked at the work that kids did last time, the kids who weren't successful weren't able to translate the problem into how can I represent it., how can I understand it, how can I draw a picture or a model that is going to show me what I need to figure out to do this problem. ...the language was a problem for them – they couldn't translate it into mathematics.</p> <p>[break in transcript]</p> <p>Teacher 19: I guess if a kid knew which information to put in which box, it would show me that they know what do they need to find out. So if they have the numbers in the wrong place, then they're confused about what the problem says. And if they have the numbers in the right place...[fades out]</p> <p>Teacher 67: So, then, our lesson is skipping [step] number three?</p>	<p><i>Jan 03, Transcript; Untranscribed interview</i></p> <p><i>Teachers hear and discuss two observations: 1) that student learning happens when they share strategies for attacking problems; 2) that the five step process is flexible enough to allow multiple strategies for solving problems [kt]. Teachers also hear why one teacher chose a particular lesson (based on her interpretation of the group's findings about student problem solving). This provides an opportunity to discuss the lesson and review their shared understanding of student difficulties [kt]. While the group seems to agree with the interpretation of student difficulties, they are unsatisfied with the chosen lesson. One teacher in particular feels the lesson is too prescriptive - it won't allow them to see how students solve the problem because they are simply asking students to fill in boxes [ls, kt].</i></p>
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	Because they're really just filling in the diagram – that's really what the lesson is about, right – one and two? And then four?	<i>One teacher shares a desire to integrate what they learned about the problem-solving guide with what they already learned about how students solve problems. She suggests that this will enable them to make sense of how students solved the previous problem, or – for students who were unable to solve the problem – where their difficulty was in the five step process [kt – ideas and data building on each other]. Teachers value classroom data on student understanding [kt, ls]. Teachers are motivated to pursue these ideas [me].</i>	<i>Jan 03, Transcript</i>
7	Teacher 62 says she would like to go back to her class and take the task they already did with their students and “put this guide to solving problems in there and see if the kids could actually break it into parts and tell me what they did to work this problem out. For the kids that didn’t get it, then I would like to see them draw a diagram and see if they can solve the problem. Because we’re going into another lesson, but we already have a lesson that not all kids succeeded in.” All teachers agree to ask their students to use the problem solving guide to solve the previous task.	<i>One teacher shares a desire to integrate what they learned about the problem-solving guide with what they already learned about how students solve problems. She suggests that this will enable them to make sense of how students solved the previous problem, or – for students who were unable to solve the problem – where their difficulty was in the five step process [kt – ideas and data building on each other]. Teachers value classroom data on student understanding [kt, ls]. Teachers are motivated to pursue these ideas [me].</i>	<i>Feb 03, Fieldnotes</i>
8	Two teachers continue to search for a lesson. One teacher finds a research lesson used in the previous year and presents this lesson to the group. She feels that the lesson focuses on steps three and four of the problem solving process - “what will you do?” and “answer the question” and this focus would elicit strategies from students. Teachers could then use that information to identify other strategies, so that students would have “more tools to choose from when solving a problem.”	<i>Lesson study materials are seen as a viable source of ideas [ls]. Teachers demonstrate continued interest in finding a lesson that will allow students to demonstrate problem solving strategies [kt, ls]. Teachers hear the rationale for the particular lesson choice, in relation to their goals [kt].</i>	<i>Feb 03, Fieldnotes</i>
9	Teacher 111 likes the lesson, but also likes the work the group had done so far – she had been using the five steps in her class. Teacher 67 likes the lesson; it fits with what they wanted to do. She doesn’t do problem solving now without going back to the five steps. Teacher 69 likes the lesson because it gives them something concrete to observe; c) it may help reveal	<i>Teachers report liking the lesson for several reasons: a) it seems to fit with the five step process; b) it gives them something concrete to observe; c) it may help reveal</i>	<i>Feb 03, Fieldnotes</i>

<p>observe. Her concern with the other lesson was that they were just going to do a lesson that taught them how to fill in boxes. Teacher 19 comments that when she was doing the other problem with her class, she was thinking about what are the different strategies that the students are using. She said she likes this current lesson because they might realize that kids have limited strategies, and they could use this lesson as a way to put new strategies into students' toolkit. She said she felt like this lesson would allow them (teachers) to see students' strategies and to see the limit of students' strategies. She also felt like the lesson really focuses students on step 3 of the problem solving sequence, and later in the unit, students could begin to identify what strategy is being used and group them.</p> <p>Teachers agree to adapt this lesson for their research lesson.</p>	<p><i>student strategies and limits to their strategies. Their comments indicate that they find the problem solving process useful and are using it [kt, me], that they are focused on student strategies [kt, ls], and that they could use data generated by this lesson to help them develop additional strategies [kt].</i></p> <p>Teachers try out the task⁴ themselves, "just to see the different strategies that we have and that might kind of open our eyes to what might happen with kids." Teachers get different answers. Teacher 111 is surprised by Teacher 67's answer. Teacher 67 explains how she got her answer.</p> <p>Teacher 111: Ohhh.</p> <p>Teacher 19: Okay, good, an ah-ha."</p> <p>Teacher 111: I didn't get that. I thought there was information that wasn't needed....</p> <p>Teacher 19: So did you come up with four buses?</p> <p>Teacher 111: I came up with four buses, but you know what, that doesn't make sense.</p> <p>Teacher 19: But this is what we want to see. What are the misconceptions kids will have when they do the problem so that we can anticipate if a child is doing that and what would the teacher reaction be....</p> <p>Teacher 111: Ohhh.</p> <p>Teacher 19: "To say, you're saying four buses, she's saying two buses,</p>
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⁴ The math task in the lesson reads as follows: "The second grade classes at _____ [school] are planning to go on a field trip. What is the least number of buses they will need if each bus has 28 seats and they hold 2 people, and one seat in the back that holds 5. Class 1: 20 students, 4 chaperones; Class 2: 20 students, 6 chaperones; Class 3: 20 students, 5 chaperones; Class 4: 20 students, 5 chaperones; Class 5: 10 students, 2 chaperones.

<p>why do you think your answers might be different?"</p> <p>Teacher 69: Teacher 111, I don't want to say that I liked that you missed on this... Many word problems are very trickily worded... You really have to pay attention to all the words. And kids have to learn to do that, I think, to be successful.</p>	<p>11 The group assigns data collection responsibilities during the research lesson.</p> <p>Teacher 19: One thing that helps in the data is to divide the group up and follow the same kids through the whole lesson. Otherwise you miss any big leaps or changes that those kids might have.</p>	<p>12 Teachers observe particular groups of students during the research lesson.</p> <p>Teacher 19: So they were done and then ...R ____ [student] said, "OK, we're done. ...Should we check it another way?" ...And I thought "Now it's really going to get interesting. I want to see how they're going to check their work." Then you stopped the class and said "Some people are finished, and if you're done early you can choose work from the back table." And I thought, "Please check it. Please check it. Please check it." [Teachers laugh.] But they didn't. They went and got another...</p> <p>Teacher 69 [observing a separate pair of students]: ...He thought it should be checked. But this was very interesting how she checked it. She didn't even look up at the top [words in the story problem]. She looked at all of her numbers to see if they were right. ...To check it, didn't even - and I thought this was really insightful - didn't mean to go back and reread it [the problem] at all. It only meant to check her addition and subtraction.</p>	<p>13 The group discusses how they might revise the instruction in the lesson plan - and in their instruction generally - to adapt to what they observed.</p> <p>Teacher 67: [responding to data that she had moved the class before they checked their work] So what I really should do when they're sharing is ask "How did you check it?" [Teachers respond "uh huh" in the affirmative.] I told you that's what I'm not good at.</p>
		<p><i>One colleague reports to colleagues what she has learned about data collection – to follow the same kids through the whole lesson [ls].</i></p>	<p><i>Feb 03, Transcript</i></p> <p><i>Teachers report data that indicated that two groups of students had unexpected problems with step 5 – they either didn't check their work at all, or they only checked their arithmetic, without rereading the problem [kt]. One teacher's data also indicate how student responses were directly related to teacher's instruction [kt].</i></p>

Teacher 19: So, that was a really interesting point for me. And I think that as teachers, that we communicate, too, to kids that it's better to do more, than to do one thing really, really thoroughly and well. ...We're so afraid of kids being done, and then "what do we do when we're done?" ...And this is always a problem when we're doing lesson study discussions, is we always build in all of these extra extensions. And I want to say "No, make them keep thinking about it."

One teacher reports that even with the five-step process, her teaching, has not supported her students to learn step 2.

Teacher 69: I always say, "...You know we're following the five steps. I don't even need to tell you to do number one and number two any more because you just will automatically do it." ...They just do it as a matter of course, on their own. Now to find out what you need to know, we of course ...look for the question marks. ...But I guess what I was finding in that one group I looked at [during the research lesson], which was A _____ and L _____, you know what, they did follow that and they were pretty successful at underlining [illustrating their understanding of steps 1 and 2], but they didn't do anything with it. So that was kind of curious to me.

Teacher 67: So many times, my kids have done questions this year and they have figured out the problem, but they haven't answered the question. Some of the questions that we've done have been trick questions, where the question is not really the solution to the math – it is using the solution to then answer the question. It isn't always the question.

Teacher 69: ...I have been guilty of that, because I've said "look for the sentence with the question mark," you know? ...I haven't done enough teaching around number two – "what do you want to find out?" Because I haven't included that piece which is, "what do you need to find out to be able to do the problem in the first place?" ...Sometimes it's not going to be on the paper. And I haven't taught that at all. ...I put down a note here, "what do they want to find out." I'm going to

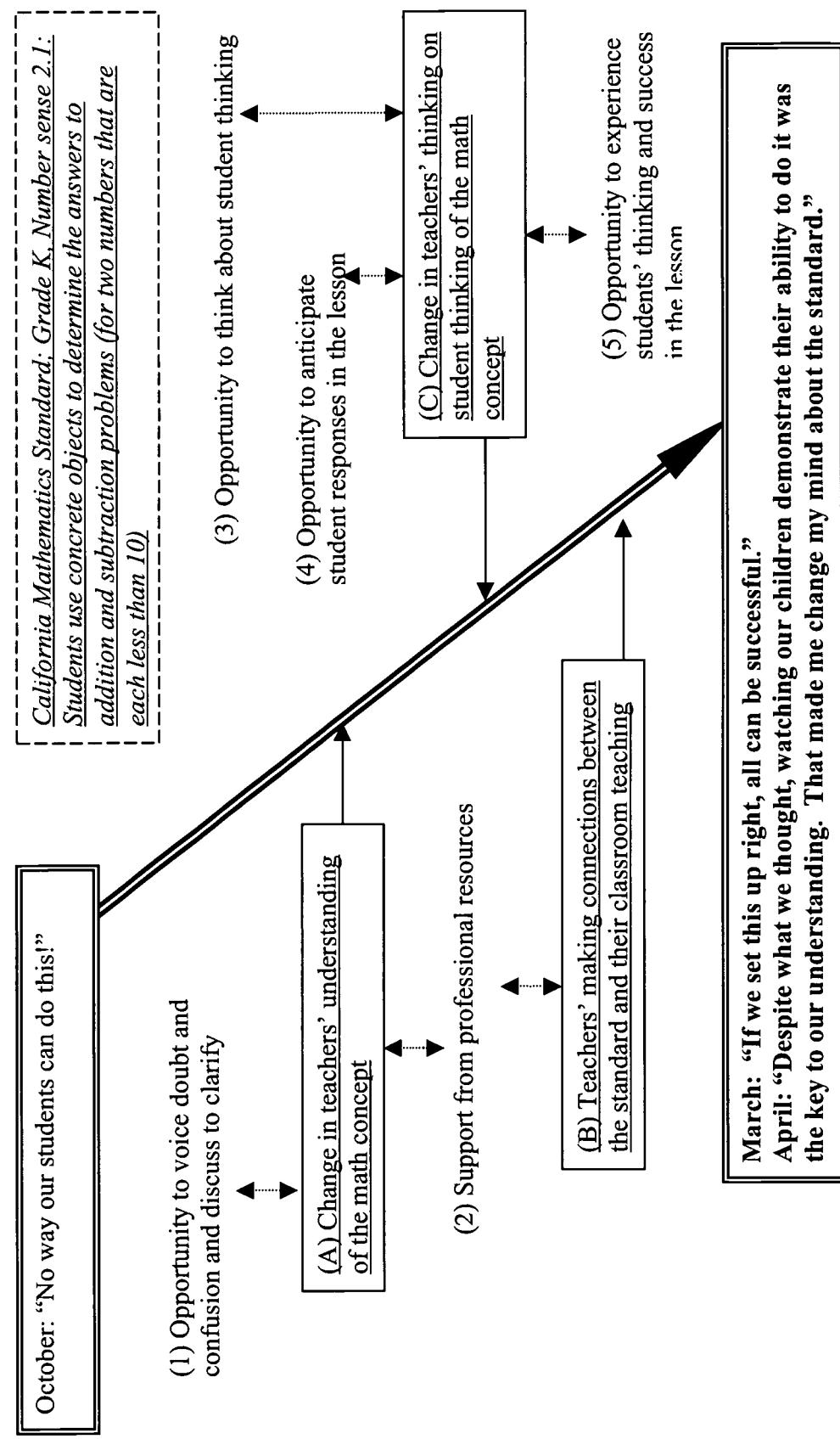
Another teacher realizes that a strategy that she is using to help students identify what they need to know in a word problem (looking for the question mark) is insufficient [lkt] Her observations and data, in combination with discussion with another teacher, indicate to her that she needs to refocus her instruction.

	<p>really do some more teaching around, and look for problems that don't... where one of the things that they want to find out doesn't have a question mark.</p>	<p>One teacher reports feeling like the problem-solving guide gives them a way of identifying students problem solving strategies and providing new strategies to help students be more flexible in their problem solving approach [kt, me]. She also reports learning about a critical mathematical skill that isn't in the curriculum, but which their review of the standard revealed [kt].</p>	<i>Feb 03, Transcript</i>
<p>14</p>	<p>Teachers discuss using the problem-solving guide more consistently from the beginning of the year.</p> <p>Teacher 19: This is really interesting for me because I have not been doing this routinely in teaching third grade. I have not been routinely taking kids through this five-step process. And now that we picked this as a standard, I mean, I would start the year next year... I'll be teaching it all year.</p>	<p>Teacher 67: I will too.</p> <p>Teacher 19: I've done word problems with kids and I've done challenging parts. But I've never, like, taken them through these five steps and realizing that it isn't in the standards, really directly in the curriculum, and yet it's a critical mathematical skill that kids need to have. Although... this has been a difficult path to go down, I'm really glad that we have because it's going to change what I'm doing. ...If I had started in September, [think] how many more strategies my kids would have now too. Like, now they just have whatever they brought from second grade. I haven't shown them new strategies. But now I'm realizing that it's an important thing – to be modeling so they have all those tools for ways to solve problems.</p> <p>Teacher 69: Yeah, and that they recognize that there are different ways to do it.</p>	<i>Mar 03, Transcript</i>
<p>15- 16</p>	<p>The lesson is taught again in another class.</p> <p>Teacher 19: ...Our group really got hung up on how many kids fit on a bus.</p>	<p>The lesson is taught in another class, providing additional data for the group to discuss [kt, ls, me]. Observations from this class reveal that students have problems with steps 1 and 2; the language of the problem seems to prohibit them from focusing on the mathematical work [kt]. Teachers are required to</p>	

<p>one side. And then the girl was arguing with him, "No, that's not what a bus looks like." So, I was really assuming that they... Come on! [Laughing] ...They've been on a school bus! You know, but you're right. They didn't have that...</p> <p>Teacher 69: I think that that kind of held them back. ...I asked some of them, how many sit on a school bus seat and some of them said three.</p>	<p>revisit the problem from the students' perspective [ls]. The data reveals that their assumption that students would be familiar with the structure of a school bus was incorrect [kt].</p> <p>Teachers revise their lesson</p> <p>Teacher 67: ...I wonder what would happen if we added a picture of one seat equals two children. Just as simple as that. ...I would like to take away that confusion that we dealt with between the number of children and the number of seats. ...If we alleviate that confusion, wouldn't it be great to see where the kids go with it.</p> <p>Teacher 19: Because the point of the lesson wasn't for them to make sense of that language.</p> <p>Teacher 67: Exactly.</p> <p>Teacher 19: The point of the lesson was to figure out the problem and use the five step problem-solving strategy to do it.</p>	<p>Teachers consider another revision to the wording of the problem to eliminate confusion for students and enable them to focus their lesson on the mathematics [kt, ls, me].</p>	<p>Mar 03, Transcript</p> <p>Teachers report that they have found a valuable tool for helping students learn how to approach multi-step problems [kt, me].</p> <p>Teachers report that they have found a valuable tool for helping students learn how to approach multi-step problems [kt, me].</p>
			<p>Teacher 62: It's really important to teach that problem solving chart and embed it all the time into everything that we do, whether it's social studies, science, math, or whatever. Because there is an organizational part... We were talking about something in the class ...and I pulled this chart out and they went like... "Oh, no. You have it too!" [Everybody laughs.] And they go "The guide to problem solving. It's in your room too?" I said, "It's going to stay in my room. And we're going to refer to this. ...You know what? It applies to every single thing. 'What do you know' and 'what do you want to find out?' And 'what will you do?' It's the same thing - answer the question. It applies to everything.</p> <p>Teacher 111: I had mine laminated... and [put into] braille. I'm so into it.</p>

Lesson Study and Teachers' Knowledge Development: Collaborative Critique of a Research Model and Methods
AERA 2003, Division K, Section 7; Session 29.058, April 22, 2003

Example #2: Changes in kindergarten Teachers' View on a Standard



Note: Double-lined textboxes show the actual quotes. Single-lined textboxes identify changes that occurred which generated the change in the teachers' discourse. Numbered sentences identify supporting opportunities for change provided by lesson study . Examples for these items are found in the data

**Lesson Study and Teachers' Knowledge Development:
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Example #2: Change in Kindergarten teachers' view of a standard

Background on the Kindergarten lesson study group

This lesson study group consisted of three kindergarten teachers with 16 to 33 years of teaching experience, most of it at the kindergarten level. The teachers had worked together in that school for many years and knew each other very well. They communicate with each other regularly and their classrooms are in close physical proximity to each other. Their first participation in lesson study occurred the previous year, when they collaborated in a group with grade 1 teachers as a part of school-wide lesson study. At that time they co-planned a grade 1 lesson, so this was their first experience planning a kindergarten research lesson. After an initial school-wide lesson study meeting in September, they met in October to make a lesson study plan for the year. They chose a new California Standard for Grade K which they felt would be very challenging to teach:

Students use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10)

Date	Data	Course of Learning
Oct.	First grade-level meeting. Agenda is to make their lesson study plan for the school year. Examine California Mathematics Standards for Grade K. Appear confused about the standard for number sense [see above]. Say "no way" the students can do it. Difficulty understanding the standard. Discuss whether it means students are to add and subtract numbers up to 10, 18, 20, or 99.	<i>Previously, teachers taught Grade K students to count up to 20, and to manipulate objects up to 5, or 10 for fast students. Their previous end-of-the-year assessment used missing-addend problems with 3, 4, and 5 as totals. New standard appears to be very advanced from prior practice. Teachers seem to feel 'trapped' in teaching the impossible to their students. (1)</i>
Nov	Teachers continue to discuss the meaning of the standard. Say they feel uncomfortable using terms "addition" and "subtraction" as they talk. In response to questions from me, they discuss activities they have done in previously in their classrooms and the relationship to this standard. Discuss whether some decomposition activities (two elephants and peanuts, fish in a double-sectioned fishbowl, birthday cake with candles with different colors) could be used to help their students with the standard.	<i>There still seemed to be some confusion about the standard, but teachers were beginning to make connections between their classroom practice and the standard. (1), (A), (B)</i>
Dec.	Interviews conducted with a sample of students (see Interview section that follows). Teachers discussed interview results.	<i>Interview results provided an opportunity for the</i>

	<p>Japanese teaching-learning trajectory for number sense/addition and subtraction was also introduced, guided their discussion.</p> <p>KW: (about interview tasks) I was surprised when I saw the questions. Could any of the students do that? I thought you would ask something more simple.</p> <p>KC: I thought it was good to do interviews like this one because they help us see student strategies. We used to just teach kids to memorize everything, but now we are saying we should see what students do, and interviews help.</p> <p>KW: Kids did well, didn't they? They did well.</p> <p>KC: (about Japanese teaching trajectory) I really liked the idea of such a sequence, and everything seems to be connected to one another.</p> <p>KW: (about using decomposition for addition and subtraction) It's totally a new concept to me, but it makes sense. It may take a while for teachers to think differently for this one.</p> <p>KC: It's too bad that we do this Alien lesson and are done with it. I wish we could do more with this. ... I will make a large alien poster for all the classes, laminate them, make multiple legs with Velcro, and work on this theme throughout the year with different numbers of legs.</p>	<p><i>teachers to think about student thinking related to this concept. Japanese teaching-learning trajectory mapped out possible paths of student learning. The teachers appeared excited as they saw what their students could do and a possible path to help them achieve the goal.. They talk about how the lesson they are planning should be extended after lesson study.</i></p> <p>(2), (3), (A), (B), (C)</p>
<p>Jan.</p>	<p>As the teachers fine-tune the lesson to be taught, they anticipate their students' responses.</p> <p>KW: What if kids come up with the combination of 5 and 0?</p> <p>KC: If they do, we'll talk about it, right? But if not, or if they don't come up with certain combination, should we tell them?</p> <p>KN: I think we should leave it up to them to decide the categories. If we give them the combination, it will be "our" category, if they say it, it will be theirs. We want this to be bottom up.</p> <p>KW: So, you're saying that we are not going to talk about 5 and 0 if the kids don't ... OK.</p> <p>KC: Some kids may have wrong numbers written after coloring.</p> <p>KN: ... (students) may color one leg using two colors, too.</p> <p>KN: These questions we have been posing would be good ones to discuss during post-lesson debriefing. These are not something we know now, but something to reflect later on.</p>	<p><i>There was no focused discussion of the standard but the teachers' comments showed that they were anticipating specific student responses now, excited as they shared what they knew about own students, and not totally giving up on teaching the standard any longer.</i></p> <p>(4), (C)</p>
<p>Feb.</p>	<p>Lesson taught three times in three classrooms.</p> <p>KW: Kids did so well, even my LD kid. She found all four combinations. I was so proud of her!</p> <p>KC: In all classes, there were more than several kids who noticed the reverse combinations, like 3 and 2 is 2 and 3. That was good.</p> <p>KC: Many kids filled out the number squares quickly. Only a few kids needed to count. Many kids reversed the numbers, though.</p> <p>KC: All kids, I mean, everyone did well. Everyone could have success. It was a good lesson.</p> <p>KC: (for future extension of the lesson) As several kids noticed, reverse combination of colors may be a good place to extend learning. Talk about different ways to make 5. And, make the</p>	<p><i>The teachers experienced their students' success. From what they anticipated that students might do, they felt that the students did extremely well. The teachers seem excited as they talked about how they would extend this lesson for their future teaching in the classrooms.</i></p> <p>(2), (3), (4), (5), (A), (B),</p>

	<p>numbers bigger than 5 and engage them in more decomposition activities.</p> <p>KW: I just saw in textbook that there are several decomposition activities. Kids are to make a train of 10 unifix cubes, break it into two parts, and record the number combinations. We will do this next week.</p>	(C)
Mar.	<p>Teachers reflected on their lesson study experiences. They filled out the reflection questionnaire individually then shared their reflections with the group.</p> <p><u>The change in their thinking related to student thinking about the math concept:</u></p> <p>KW: If you set this up right, all can be successful</p> <p>KC: I think this experience is key – a great way to introduce concept of joining together before formal $2 + 3 = 5$.</p> <p>KN: Students don't necessarily transfer the idea of "combinations" over to "addition" at this early stage.</p> <p>KC: When I first thought this, I thought NO WAY! But after alien lesson, this makes sense. Our kids can do this.</p> <p>KW: Students don't need to know $9 + 9$ is 18, right? As long as they can put two 9s together and know that is 18. They can do it that way. This standard makes better sense to me now.</p> <p>KC: I feel that I can teach the textbook chapter on this now, too. It's not THAT impossible. Students can do more difficult tasks than I first thought.</p> <p>KC: They (students) are not as impossible as I once thought – developing these very motivating lessons helps get to the target.</p> <p><u>The change in their understanding of the math concept:</u></p> <p>KW: Idea of embedded number 5</p> <p>KN: What's changed is seeing new ways to discuss and teach this concept</p> <p>KC: I just never thought about it this way, like number combinations and embedded numbers. It makes sense now. If you present the way we did, it all makes sense. Also, in the other meeting (PD meeting), ... (we learned) how to subtract numbers using decomposition. I really wish that I learned it that way when I was little. All I knew was to carry and cross out, and that was hard. I wish I knew it that way ...</p> <p><u>Connection to the future teaching of the standard:</u></p> <p>KW: We need to work on higher numbers up to 18</p> <p>KC: I have moved on to the textbooks for combination to 5, nice flow from experience</p> <p>KN: Use idea of moveable creatures or other objects to create an easy tactile way to learn number concepts. Also change numbers (3, 5, 7, etc.)</p> <p>KC: (Textbook) chapter has number combinations. It teaches 3 and 1 more is 4, 4 and 1 more is 5, etc., so I jumped to that chapter and</p>	<p><i>Teachers expressed confidence and excitement about teaching the standard. They appeared to feel good about teaching this and know their students can do it. They seemed to have developed a new way to look at and think about student learning of the concept. They repeatedly stated how good it was to see their students being successful.</i></p> <p>(A), (B), (C)</p>

	<p>started it. Nice tie in to the chapter.</p> <p>KW: Perhaps (next year), start in November, with ghost and pumpkin theme . Students can start that way, and start decomposing earlier.</p>	
Apr.	<p>Reflection is on meeting agenda. Teachers discuss the changes in their thinking about the standard.</p> <p>KW: We never had to push them up to 18, 10 maybe, even for the end-of-the-year assessment, we went up to 5. ... We never had to go up to 20, pushing two sets of things together. At the beginning of the year, the standard sounded like there was nowhere to go. ... So, how are they going to do this?</p> <p>KC: But we did up to 5 for the lesson study, and we really liked to do this so that everyone could be successful. ... they are concrete-operational, if they can get hands on something, they can go up to the higher numbers as long as they can touch it. They can go up quite high. ... Once I found out that it was the standard we had to live with, I found many creative ways to do it, like making it in a song, or things</p> <p>KW: Pushing two sets of things together, for some of them, there is a lot of room for error. And, doing it all the way up to 18 is challenging, especially for our lower ones. For some, though, it's not a problem at all.</p> <p>KN: Despite what we thought, watching our children demonstrate their ability to do it is the key to our understanding. That made me change my mind about the standard.</p> <p>KW: Everyone was able to do it, and I was so happy about that. second language kids can do it, special ed kids can do it.</p> <p>KN: The nicest thing for me was that I was not too thrilled about the way Kindergarten math was changing, how they have so much emphasis on workbooks and worksheets. So, it was very nice to do something like lesson study. Like, now we are stuck with teaching the standard, but we don't necessarily do it the way they are telling us to do it (with worksheets and workbooks). As long as we can teach to the standard, we can do Aliens and other things, too.</p>	<p><i>Teachers comment that seeing the students' success was the key to their understanding of student thinking and the standard. Teachers seem to feel they own the lesson and the standard now. Initially, the standard could not be connected to their everyday teaching. Lesson study provided opportunities for them to share their confusion, discuss different interpretations of the standard, and anticipate their students' responses to specific tasks related to the standard. The Japanese teaching-learning trajectory also provided a connection to practice. The teachers are motivated to teach the standard for the next year.</i></p> <p>(2), (5), (A), (B), (C)</p>

Questions and Issues:

1. How do we organize the data to effectively uncover and illustrate teacher learning?
2. What additional data would be helpful?

Kindergarten Student Interview Summary

Background: Student interviews were conducted to assess student thinking, their understanding of quantities, and to trace the changes over time. A representative sample of students was selected from each of the three classrooms to be interviewed individually (selection was done by classroom teachers). Students have been interviewed twice so far this year: December (before the research lesson was taught) and April (after the research lesson was taught). Fifteen (15) students were interviewed initially in December, but only 12 were interviewed in April (one had moved away, one was absent, one was not tape-recorded correctly). The interviews were tape-recorded, and the tapes were transcribed. The interviewer also took interview notes during the interviews. The data on the 12 students were analyzed longitudinally as a group, while the data on students who had only one data-point (3 in December, 5 in April) were also analyzed to compare the general patterns. For individual interviews, students were asked three questions: 1) "What two numbers make 5 together?", 2) $2 + 6$ ("If we have 2 things, and we have 6 things, how many do we have together?"), and 3) $8 - 2$ ("If we have 8 things, and we take 2 things away, how many things do we have?"). When appropriate, students were also asked to identify two numbers to make 10 and 18, $6 + 7$, and $13 - 6$. After they stated the answer, they were asked to describe the strategy they used to get the answer. Student responses were coded according to Conception of Quantities Levels (see the following page).

Summary of Interview Results:

Problems	Responses	December	April
"What two numbers make 5 together?	Incorrect response	50%	25%
	Identify correct pair	42%	0
	Identify more than one correct pair	8%	75%
$2 + 6$ ("If we have 2 things, and we have 6 things, how many do we have together?")	Incorrect response*	33%	42%
	Pre-Level I (need support to count all)	33%	8%
	Level I (count all)	25%	8%
	Level II (count on & finger recognition)	0	17%
	Recall	8%	25%
$8 - 2$ ("If we have 8 things, and we take 2 things away, how many things do we have?")	Incorrect response*	25%	25%
	Pre-Level I (need support to count all and take away)	42%	0
	Level I (count all and take away)	17%	17%
	Level II (count down & finger recognition)	8%	42%
	Recall	8%	17%

Incorrect response*: The types of incorrect responses for addition and subtraction questions changed from December to April. In December, many students solved problems incorrectly due to the lack of one-to-one correspondence and giving up (not wanting to try). In April, most incorrect responses were due to incorrect recall attempt.

Conceptions of Quantities Levels of Addition and Subtraction Solution Methods
 (Adapted from Fuson, 1992a, 1992b; Fuson and Kwon, 1992; Murata and Fuson, under review)

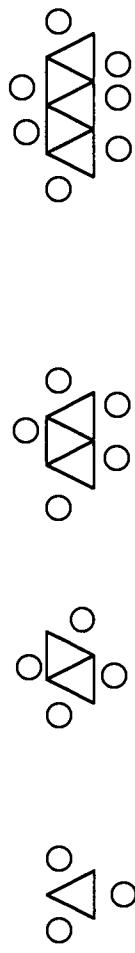
Level	Conception of Quantities	Addition Solution Methods	Subtraction Solution Methods
I	Perceptual unit items Single presentation of the addend or the sum at a time.	<u>Count All</u> For $8 + 6$, count “ <u>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14</u> .” (may also count one or both addends first)	<u>Count All and Take Away</u> For $14 - 8$, count 14 objects, then take away 8 objects, then count the remaining 6 objects.
II	Sequence unit items Simultaneous presentation of an addend within the sum.	<u>Count On</u> For $8 + 6$, count on from 8 six words “ <u>8, 9, 10, 11, 12, 13, 14</u> , 14 is my total.”	<u>Count Down</u> For $14 - 8$, count 8 words down from 14, “ <u>14, 13, 12, 11, 10, 9, 8, 7, 6, 5</u> are left.”
III	Ideal chunkable unit items Simultaneous embedded mental presentations of both addends and the sum: One addend is chunked to relate to a known addition or subtraction	<u>Derived Facts/Recomposing (U.S. students)</u> <u>Doubles</u> For $7 + 6$, use the known double, $6 + 6 = 12$, then think of $7 + 6$ as 1 more than $6 + 6$, therefore $12 + 1 = 13$.	<u>Doubles</u> For $15 - 7$, use the known double, $7 + 7 = 14$, then think of $15 - 7$ as 1 more than $14 - 7$, therefore $15 - 7 = 8$.
		<u>Make a ten from one number (Asian students)</u> <u>Up-Over-Ten</u> For $8 + 6$, think 8 and 2 more is 10, then take the 2 from 6 ($6 - 2 = 4$), and add it to 10 equals 14 (ten-four).	<u>Take-From-Ten</u> For $14 - 8$, think 8 and 2 make 10, then add the 2 to 4 ($2 + 4 = 6$) equals 14 (ten four).
		<u>Break-Apart-to-Make-Ten</u> For $8 + 6$, think 8 and 2 make 10, separate 6 into 2 and 4, add 2 and 8 to make 10, add 4 more to get 14.	<u>Down-Over-Ten</u> For $14 - 8$, think 8 as $4 + 4$, take one 4 from 14 down to 10 ($14 - 4 = 10$), then take other 4 down from 10 ($10 - 4 = 6$), 6.
		<u>Make a ten from fives within each number</u> <u>Add Fives and Then Add Amounts Over Five</u> For $8 + 6$, think 8 as $5 + 3$, 6 as $5 + 1$, then $5 + 5 = 10$ and $3 + 1 = 4$, and $10 + 4 = 14$.	

NOTE: Recall happens increasingly through the levels, and it varies with individuals.

Example 3: Videotape from Summer Institute

The videotape shows an ad hoc lesson study group of six elementary teachers (from different schools) who worked together to plan, teach, revise, and re-teach a research lesson, during a 2-week summer institute in algebra and lesson study. Their charge was to focus on an aspect of elementary mathematics that provides a foundation for algebra. They studied state standards and existing curricula (including the district's newly-adopted textbook) and decided to focus their research lesson on students' identification and mathematical representation of patterns, using a textbook lesson as the basis for their research lesson. In the lesson, students identify and mathematically represent as a rule the number of seats that fit around a row of triangle tables, arranged in a parallelogram (see illustration).

We have a long skinny room and triangle tables that we need to arrange in a row with their edges touching, as shown. Assuming each side can hold one seat, how many seats will 1 table, 2 tables, 3 tables hold? Is there a pattern that helps you figure out how many seats 10 tables will hold?



12

# Tables	# Seats
1	3
2	4
3	5
4	
5	

Video Transcript
Edited Segment 1

Planning, August 7th, 2002

Teacher 1: I thought when we added a triangle we are adding two. But the output chart here is adding one and I am not, I don't understand why that is because you showed

Teacher 6: I just counted them. I.

Teacher 5: because the third one is now a combined one

Teacher 2: One plus two two plus two this way.

Teacher 1: Oh.

Teacher 5: So maybe it would be good time to have us do the activity

Teacher 1: Please

Teacher 6: Because if you have one triangle you have three. That plus two. One of those three becomes a

Researcher Notes

This edited segment includes clips from two planning meetings for the group's upcoming research lesson. At the first meeting, teacher 1 notices from the output chart that adding one triangle adds one seat, and she says this differs from what she thought about the problem. (She thought that adding a triangle added two new seats).

Teacher 1's capacity to express her puzzlement: What does it say about her/the group in terms of motivation, efficacy, collaborative skills or other aspect of lesson study capacity?

<p>combined.</p> <p>Teacher 1: You have one triangle equal to three.</p> <p>Teacher 4: You are not counting them.</p> <p>Teacher 6: You share it</p> <p>Teacher 1: Two of them become combined. That's why. You don't have five. So I'm thinking how come I don't have two, three plus two.</p> <p>Teacher 4: So you don't count the shared side. Is that what it is. You don't count the shared side.</p> <p>Teacher 5: It is the number of triangles plus two.</p> <p>Teacher 2: plus two</p> <p>Teacher 5: Number of triangles then here is the plus two goes this way. So that's what one of the ah ha's that people had yesterday, was when the kids look at the patterns we look this way instead of looking the across way.</p> <p>Teacher 1: I was just doing that. That's what Cathy was saying</p> <p>Teacher 5: Right. You were looking the down way instead of seeing the pattern also going this way.</p> <p>Teacher 5: Do we have enough triangles here for people to try to play, do it themselves and keep adding triangles and create your own chart just to kind of...??? Do that.</p> <p>Teacher 5: See how it feel for kids.</p> <p>Teacher 1: We have enough. I am still though. I am still, why isn't it, if I add a triangle</p> <p>Teacher 4: You are only counting the outside. You are not counting it.</p> <p>Teacher 1: Why am I not, three, so there is the two.</p>	<p>Group members offer explanations ("because the third one is now a combined one"; one of those three becomes combined") <i>kt</i></p> <p>Group members use manipulatives to solve the problem themselves <i>ls kt</i></p>	<p>Teacher 5 connects their own difficulty seeing plus two pattern to students' difficulty <i>kt</i></p> <p><i>ls</i></p> <p><i>kt</i></p>
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Planning, August 9th, 2002

Teacher 1: Okay students will discover a pattern and they will represent the pattern as a rule.

Teacher 5: That's the plus two. So that's we are okay there.

Teacher 1: They will understand what a mathematical rule is and will be introduced to the idea of representing the rule as an equation.

Teacher 1 describes plus two pattern clearly at a later planning meeting in the

<p>Teacher 5: So representing the rule as an equation that's a little bit umm Teacher 2: ... the direction.</p> <p>Teacher 1: But it is an equation. We are saying the number of tables plus two equals the number of chairs or number of seats. That is where we want them to get them to after the end of the easel time</p>	<p>course of a goals discussion. Evidence that she understands the pattern now. <i>Is this strong evidence? Ideally, what other evidence would be useful?</i></p> <p>Edited Segment 2</p> <p>Planning, August 7th, 2002</p> <p>Teacher 6: I second what Teacher 3 says about, I think the facilitator's role is to stop, make sure you are on the process and make sure that everybody's, you know everybody's opinion is counted, you know.</p> <p>Teacher 5: hmm. So maybe we are hearing that too that facilitator needs to be a little bit more aggressive, a little bit you know more in there, saying let's slow down, let's poll everybody, let's say what we are doing right now. Would you feel more comfortable with that?</p> <p>Planning, August 8th, 2002</p> <p>Teacher 5: So this would be a good place for us to anticipate what we think is going to happen, misconceptions that might happen when they do 4, 5, and 6</p> <p>Teacher 1: Okay. But first let's hear from everybody I think, because we had kind of a proposal on the table and I think one of the things that happen yesterday was we would have a proposal and we sort of assumed everyone was on board, but we weren't. Is everybody on board with this, this is</p> <p>... that is why</p> <p>Teacher 1: So we have a clear understanding on this</p> <p>... teacher guided</p> <p>... 1, 2, 3</p> <p>Teacher 1: Teacher guided 1, 2, 3, independent 4 through 6</p> <p>Teacher 2: Okay</p> <p>Teacher 4: And everybody's okay with this.</p>
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